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UNIVERSITY OF MUMBAI

No. UG/ 133 of 2017

CIRCULAR:-

A reference is invited to the Syllabi relating to the B.A / B.Sc degree course, vide this office Circular No. UG/90 of 2012-13, dated 8th November, 2012 and the Principals of the affiliated Colleges in Arts & Science are hereby informed that the recommendation made by Board of Studies in Mathematics at its meeting held on 27th March, 2017, has been accepted by the Academic Council at its meeting held on 11th May, 2017 vide item No. 4.193 and that in accordance therewith, in revised syllabus as per the (CBCS) for S.Y.B.A / S.Y.B.Sc (Mathematics) (Sem III & IV) which is available on the University's website (www.mu.ac.in) and that the same has been brought into force with effect from the academic year 2017-18.

MUMBAI – 400 032 REGISTRAR

31st July, 2017

To,

The Principal of the affiliated Colleges in Arts / Science and the Head of Recognized Institutions concerned.

A.C/4.193/11.05.2017

No. UG/133 -A of 2017 MUMBAI-400 032 31st July, 2017

Copy forwarded with compliments for information to :-

1) The Co-ordinator, Faculty of Arts / Science,
2) The Offg. Director, Board of Examinations and Evaluation,
3) The Director, Board of Students Development,
4) The Professor-cum-Director, Institute of Distance and Open Learning.
5) The Co-Ordinator, University Computerization Centre.

REGISTRAR

...PTO.

UNIVERSITY OF MUMBAI

Syllabus

for

S.Y.B.A./S.Y.B.Sc. (CBCS)

Program: B.A/B.Sc.

Course: Mathematics

with effect from the academic year 2017-2018

Preamble

The Board of Studies in Mathematics has prepared the syllabus of S.Y.B.A./S.Y. B.Sc. (w.e.f. 2017-18) and T.Y.B.A./T.Y. B.Sc. (w.e.f. 2018-19) in the subject of Mathematics under the Choice Based Credit System (CBCS).

The syllabus provides best learning experience to the students as well as to the teachers by offering

1. two interdisciplinary courses in Semesters III and IV of S.Y.B.A./S.Y. B.Sc. and
2. two projects based courses in Semesters V and VI of T.Y.B.A./T.Y. B.Sc.

The interdisciplinary course offered in Semesters III is INTRODUCTION TO COMPUTING AND PROBLEM SOLVING - I. The Aim of this course is to develop Algorithm thinking towards problem solving.

The interdisciplinary course offered in Semesters IV is INTRODUCTION TO COMPUTING AND PROBLEM SOLVING - II. In this course students are enabled to write their own Programs in Python.

In the two project courses offered in Semesters V and VI of T.Y.B.A./T.Y. B.Sc. a student can do a project on a topic from Mathematics, Financial Mathematics, Statistics, and Computer programming. Nearly fifty topics are listed for this project courses in this syllabus.

By this syllabus, the quality of education offered to students is enhanced. This curriculum creates positive improvements in the educational system.

The curriculum retains the current workload of Mathematics Departments.

S.Y.B.A./S.Y.B.Sc. (CBCS)
Semester III

CALCULUS III				
Course Code	Unit	Topics	Credits	L/W
USMT301,UAMT301	Unit I	Functions of several variables	2	3
	Unit II	Differentiation		
	Unit III	Applications		
ALGEBRA III				
USMT302, UAMT302	Unit I	Linear transformations and Matrices	2	3
	Unit II	Determinants		
	Unit III	Groups, Subgroups		
INTRODUCTION TO COMPUTING AND PROBLEM SOLVING - I				
USMT303	Unit I	Algorithms	2	3
	Unit II	Graphs & The shortest path algorithm		
	Unit III	Trees & Traversal algorithm		
PRACTICALS				
USMTP03		Practicals based on USMT301,USMT302 and USMT303	3	5
UAMTP03		Practicals based on UAMT301,UAMT302	2	4

Teaching Pattern

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per course (the batches to be formed as prescribed by the University). Each practical session is of 48 minutes duration.

S.Y.B.A./S.Y.B.Sc. (CBCS)
Semester IV

CALCULUS IV				
Course Code	Unit	Topics	Credits	L/W
USMT401,UAMT401 USMT401,UAMT401	Unit I	Nested Interval Theorem & Applications	2	3
	Unit II	Riemann integration		
	Unit III	Indefinite and improper Riemann integrals, Double integrals		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT402, UAMT402	Unit I	First order first degree differential equations	2	3
	Unit II	Second order linear differential equations		
	Unit III	Linear system of ODEs		
INTRODUCTION TO COMPUTING AND PROBLEM SOLVING - II				
USMT403	Unit I	Problem solving strategies	2	3
	Unit II	Python programming language		
	Unit III	Iterations, Strings & File Handling in Python		
PRACTICALS				
USMTP04		Practicals based on USMT401,USMT402 and USMT403	3	5
UAMTP04		Practicals based on UAMT401,UAMT402	2	4

Teaching Pattern

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per course (the batches to be formed as prescribed by the University). Each practical session is of 48 minutes duration.
3. For the Practicals of USMT403, Python version 2.7.9 shall be used by all colleges.

<u>Proposed Syllabus for Semester III & IV</u>

SEMESTER III

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

USMT301/UAMT301 CALCULUS III

Unit I: Functions of several variables (15 Lectures)

The Euclidean inner product on $\mathbb{R}^n$ and Euclidean norm function on $\mathbb{R}^n$, distance between two points, open ball in $\mathbb{R}^n$, definition of an open subset of $\mathbb{R}^n$, neighbourhood of a point in $\mathbb{R}^n$, sequences in $\mathbb{R}^n$, convergence of sequences- these concepts should be specifically discussed for $\mathbb{R}^2$ and $\mathbb{R}^3$.

Functions from $\mathbb{R}^n$ to $\mathbb{R}$ (scalar fields) and functions from $\mathbb{R}^n \longrightarrow \mathbb{R}^m$ (vector fields), limits and continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector field.

Directional derivatives and partial derivatives of scalar fields.

Mean value theorem for derivatives of scalar fields.
Reference for Unit I:
Sections 8.1, 8.2 8.3 8.4,8.5,8.6, 8.7, 8.8, 8.9, 8.10 of *Calculus, Vol. 2* (Second Edition) by T. Apostol.

Unit II: Differentiation (15 Lectures)
Differentiability of a scalar field at a point of $\mathbb{R}^n$ (in terms of linear transformation) and on an open subset of $\mathbb{R}^n$, the total derivative, uniqueness of total derivative of a differentiable function at a point, simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2, f(x, y, z) = x + y + z$, differentiability at a point of a function f implies continuity and existence of direction derivatives of f at the point, the existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.

Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.

Chain rule for scalar fields.

Higher order partial derivatives, mixed partial derivatives, sufficient condition for equality of mixed partial derivatives.
Reference for Unit II:
Sections 8.11, 8.12 8.13 8.14,8.15,8.16, 8.17, 8.23 of *Calculus, Vol. 2* (Second Edition) by T.

APOSTOL.

Unit III: Applications (15 Lectures)
Second order Taylors formula for scalar fields.

Differentiability of vector fields, definition of differentiability of a vector field at a point, Jacobian matrix, differentiability of a vector field at a point implies continuity. The chain rule for derivative of vector fields (statements only).

Mean value inequality.

Hessian matrix, Maxima, minima and saddle points.

Second derivative test for extrema of functions of two variables.

Method of Lagrange multipliers.
Reference for Unit III:
Sections 8.18, 8.19, 8.20, 8.21, 8.22, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 of
Calculus, Vol. 2 (Second Edition) by T. APOSTOL.

Recommended Text Books:

1. T. APOSTOL: *Calculus, Vol. 2*, John Wiley.
2. J. STEWART, *Calculus*, Brooke/Cole Publishing Co.

Additional Reference Books

1. G.B. THOMAS and R. L. FINNEY, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley, 1998.
2. SUDHIR. R. GHORPADE and BALMOHAN V. LIMAYE, *A Course in Calculus and Real Analysis*, Springer International Edition.
3. HOWARD ANTON, *Calculus - A new Horizon*, Sixth Edition, John Wiley and Sons Inc, 1999.

USMT302/UAMT302 ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit I: Linear transformations and Matrices (15 Lectures)
Linear transformations, representation of linear maps by matrices and effect under a change of basis, examples.

Kernel and image of a linear transformation, examples. Rank-Nullity theorem and applications.

Composite $S \circ T$ of linear maps $T : V \longrightarrow W$ & $S : W \longrightarrow U$ of f.d. real vector spaces V, W, U

and matrix representation of $S \circ T$.

Linear isomorphisms, inverse of a linear isomorphism. Any n-dimensional real vector space is isomorphic to $\mathbb{R}^n$.

The following are equivalent for a linear map $T : V \longrightarrow V$ of a finite dimensional real vector space:

1. T is an isomorphism.
2. ker$T = \{0\}$.
3. Im$(T) = V$.

Recommended Text Book for Unit I:
S. KUMARESAN, *Linear Algebra A Geometric Approach*, PHI, 2014 (*sections 4.1,4.2, 4.3, 4.4*)

Unit II: Matrices and Determinants (15 Lectures)

The matrix units, row operations, elementary matrices. Elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

Row space and column space of a matrix, row rank and column rank of a matrix, equivalence of the row and the column rank, invariance of rank upon elementary row or column operations.

$\mathbb{R}^n$ is the space of column vectors $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ where each $x_j \in \mathbb{R}$, equivalence of rank of an $n \times n$-matrix and rank of the linear transformation $L_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ $(L_A(x) = Ax \ \forall \ x \in \mathbb{R}^n)$, the dimension of solution space of the system of linear equations $Ax = 0$ equals $n -$ rank(A). The solutions of non-homogeneous systems of linear equations represented by $Ax = b$; existence of a solution when rank$(A) =$ rank(A, b); the general solutions of the system is the sum of a particular solution of the system and the solutions of the associated homogeneous system.

Determinant $D(A^1, A^2)$ of order 2 and its properties:

1. As a function of column vectors, the determinant is linear.
2. If the two columns are equal, then the determinant is equal to 0.
3. If I is the unit matrix, $I = (E^1, E^2)$, then $D(E^1, E^2) = 1$.

Results on Determinants of order 2 :

1. If one adds a scalar multiple of one column to the other, then the value of the determinant does not change.
2. The determinant of A is equal to the determinant of its transpose.
3. Two vectors A^1, A^2 of $\mathbb{R}^2$ are linearly dependent if and only if the determinant $D(A^1, A^2) = 0$.

4. Let ϕ be a function of two variables $A^1, A^2 \in \mathbb{R}^2$ such that ϕ is bilinear (i.e. ϕ is linear in each variable), $\phi(A^1, A^1) = 0 \ \forall \ A^1 \in \mathbb{R}^2$ and $\phi(E^1, E^2) = 1$ where $E^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, E^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the standard unit vectors of $\mathbb{R}^2$, then $\phi(A^1, A^2)$ is the determinant $D(A^1, A^2)$.

Determinants of order $3\times 3, n\times n$, expansion of the determinant according to i-th row, properties of the determinant function.

Results (without proof): For two $n \times n$ matrices $A\&B$, $\text{Det}(A) = \text{Det}({}^tA)$, $\text{Det}(AB) = \text{Det}(A)\text{Det}(B)$.

Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants, the existence and uniqueness of the system $Ax = b$ where A is an $n \times n$-matrix with $\det(A) \neq 0$.

Cofactors and minors, adjoint $\text{adj}(A)$ of an $n \times n$-matrix A, $A\,\text{adj}(A) = \det(A)\text{Id}$ (without proof). A $n \times n$-real matrix is invertible if and only if $\det(A) \neq 0$ and $A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$ for an invertible matrix A. Cramer's rule.

Determinant as area and volume.
Recommended text book for Unit II:
Introduction to Linear Algebra by Serge Lang.

Unit III: Groups & subgroups (15 Lectures)
Definition of a group, Abelian group, Order of a group, finite groups, infinite groups.
Examples of groups including:

1. $\mathbb{Z}, \mathbb{R}, \mathbb{C}$ under addition.
2. $\mathbb{Q}^*(= \mathbb{Q} \setminus \{0\}), \mathbb{R}^*(\mathbb{R} \setminus \{0\}), \mathbb{Q}^+(=$ positive rational numbers$), \mathbb{C}^*$ under multiplication.
3. $\mathbb{Z}_n$ (=the group of residue classes modulo n) under addition.
4. U_n (= the group of prime residue classes modulo n) under multiplication
5. S_n (=the group of all permutations of $\{1, 2, \cdots, n\}$).
6. Klein 4-group.
7. The group of symmetries of a plane figure. The Dihedral group D_n (= the group of symmetries of a regular polygon of n sides in the plane $\mathbb{R}^2$ (n=3,4)) under composition.
8. $M_{m\times n}(\mathbb{R})$ (=the group of all $m\times n$-matrices with real entries) under addition of matrices.
9. $GL_n(\mathbb{R})$ (=the group of invertible $n \times n$ matrices with real entries) under multiplication of matrices.

Subgroups, Cyclic groups:

1. S^1 is a subgroup of $\mathbb{C}^*$, μ_n is a subgroup of S^1.

2. Cyclic groups (examples of $\mathbb{Z}, \mathbb{Z}_n, \mu_n$) and cyclic subgroups.
3. The center $Z(G)$ of a group G as a subgroup of G.
4. Cosets, Lagrange's theorem.

Group homomorphisms, and isomorphisms. Examples and properties. Automorphism of a group and inner automorphisms.

Recommended Text Books for Unit III:

1. I.N. HERSTEIN, *Topics in Algebra*, Vikas Publishing House.
2. J.B. FRALEIGH, *A first course in Abstract Algebra*, third edition, Narosa, New Delhi.

Additional Reference Books:

1. M. ARTIN: *Algebra,* Prentice Hall of India Private Limited.
2. K. HOFFMAN AND R. KUNZE, *Linear Algebra,* Tata McGraw-Hill, New Delhi.
3. G. STRANG, *Linear Algebra and its applications,* International Student Edition.
4. L. SMITH, *Linear Algebra,* Springer Verlag.
5. A. RAMACHANDRA RAO AND P. BHIMA SANKARAN, *Linear Algebra,* Tata McGraw-Hill, New Delhi.
6. T. BANCHOFF AND J. WERMER, *Linear Algebra through Geometry,* Springer Verlag New York, 1984.
7. SHELDON AXLER, *Linear Algebra done right,* Springer Verlag, New York.
8. KLAUS JANICH, *Linear Algebra.*
9. O. BRETCHER, *Linear Algebra with Applications,* Pearson Education.
10. G. WILLIAMS, *Linear Algebra with Applications,* Narosa Publication.

USMT303 INTRODUCTION TO COMPUTING AND PROBLEM SOLVING - I

Aim of this course:

1. *to develop Algorithm thinking towards problem solving and*
2. *to study Problem solving strategies like divide and concur, recursive thinking etc.*

Unit I: Algorithms (15 Lectures)

A. Definition of an algorithm, characteristics of an algorithm, Selection and iterative constructs in pseudocode, simple examples such as

   (a) Exchanging values of variables,
   (b) Sum of n given numbers.

B. Searching and sorting algorithms including the following:
   (a) Finding maximum and/or minimum element in a finite sequence of integers,
   (b) The linear search and binary search algorithms of an integer x in a finite sequence of distinct integers,
   (c) Sorting of a finite sequence of integers in ascending order,selection sort.

C. Algorithms on integers:
   (a) Modular exponent,
   (b) Euclidean algorithm to find the g.c.d of two non-zero integers.

D. Complexity of algorithm: Big O notation, Growth of functions, Time complexity, Best case, Average case, Worst Case complexity. Using big O notation to express the best, average and worst case behaviour for sorting and searching algorithms.

E. Recursion, Examples including:
   (a) Tower of Hanoi
   (b) Fibonacci sequence

Reference for Unit I:
Chapter 3 of Discrete Mathematics and Its Applications by Kenneth H. Rosen, (McGraw Hill, seventh Edition).

Unit II: Graphs (15 Lectures)

A. Introduction to graphs: Types of graphs: Simple graph, directed graph, (One example/graph model of each type to be discussed).

B. (a) Graph Terminology: Adjacent vertices, degree of a vertex, isolated vertex, pendant vertex in a undirected graph.
   (b) The handshaking Theorem for an undirected graph (statement only), Theorem: An undirected graph has an even number odd vertices (statement only).

C. Some special simple graphs (by simple examples): Complete graph, cycle, wheel in a graph, Bipartite graph, regular graph.

D. Representing graphs and graph isomorphism:
   (a) Adjacency matrix of a simple graph.
   (b) Incidence matrix of an undirected graph.

E. Connectivity:
   (a) Paths, circuits, simple paths, simple circuits in a graph (simple examples).
   (b) Connecting paths between vertices (simple examples).

(c) Euler paths and circuits, Hamilton paths and circuits, Diracs Theorem (statement only), Ores Theorem (statement only)

(d) Planar graphs, planar representation of graphs, Eulers formula. Kuratowskis Theorem (statement only).

F. **Algorithms**:
Shortest path problem: Construction of Eulerian path by Fleury's Algorithm, The shortest path algorithm - Dijkstras Algorithm, Floyd's Algorithm to find the length of the shortest path.

Reference for Unit II:
*Sections 10.1, 10.2, 10.3, 10.4, 10.5, 10.6, 10.7 of Chapter 10 of Discrete Mathematics and Its Applications*by KENNETH H. ROSEN, (McGraw Hill Edition, seventh Edition).

Unit III: Trees (15 Lectures)

A. (a) Trees: Definition and Examples.
   (b) Forests, binary trees
   (c) Trees as models.
   (d) Properties of Trees (no proofs).

B. Application of Trees:
   (a) Binary Search Trees, Algorithm for locating an item in or adding an item to a Binary Search Tree.
   (b) Decision Trees (simple examples).
   (c) Algorithm for Huffman's coding, construction of Huffman's code by examples.

C. Minimum Spanning Trees, Prims Algorithm, Kruskals Algorithm (The Proofs of the results in this unit are not required and may be omitted).

Reference for Unit 3:
Chapter 9, Sections 9.1, 9.2, 9.3, 9.4, 9.5 of Discrete Mathematics and Its Applications by KENNETH H. ROSEN (McGraw Hill Edition).

Recommended Text Books:

1. R.G. DROMEY, *How to Solve it by computers*, Prentice-Hall India.
2. R. WILSON, *Introduction to Graph theory*, Fourth Edition, Prentice Hall.
3. T. H. CORMEN, CHARLES E. LEISENON and RONALD L. RIVEST: *Introduction to Algorithms*, Prentice Hall of India, New Delhi, 1998 Edition.
4. K. H. ROSEN, *Discrete Mathematics and Its Applications*, McGraw Hill Edition.
5. B. KOLMAN, ROBERT BUSBY, SHARON ROSS: *Discrete Mathematical Structures*, Prentice-Hall India.

6. N. Biggs, *Discrete Mathematics*, Oxford.

Additional Reference Books:

1. D. B. West, *Introduction to graph Theory*, Pearson.
2. F. Harary, *Graph Theory*, Narosa Publication.
3. Graham, Knuth and Patashnik, *Concrete Mathematics*, Pearson Education Asia Low Price Edition.

USMTP03/UAMTP03 Practicals

A. Practical for USMT301/UAMT301:

(1) Sequences in $\mathbb{R}^2, \mathbb{R}^3$, limits and continuity of scalar fields and vector fields using 'definition' and otherwise, iterated limits.

(2) Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.

(3) Total derivative, gradient, level sets and tangent planes.

(4) Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.

(5) Taylor's formula, differentiation of a vector field at a point, finding Hessian/Jacobean matrix, Mean value inequality.

(6) Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.

1. Miscellaneous Theoretical Questions based on full paper.

B. Practical for USMT302/UAMT302:

(1) Rank-Nullity Theorem.

(2) System of linear equations.

(3) Computation of row rank and column rank of 3×3 matrices.

(4) Calculating determinants of matrices, triangular matrices using definition and Laplace expansion.

(5) Finding inverses of matrices using adjoint.

1. Groups, Subgroups, Lagranges Theorem, Cyclic groups and Groups of Symmetry.

2. Group homomorphisms, isomorphisms.
3. Miscellaneous Theoretical Questions based on full paper.

C. Practical for USMT303:

1. Describe an algorithm to count total number of positive and negative values from the set $\{23, -632, 325, 63, -63, 0, -55, -652.23, 65.21, 98.235, -1\}$.
2. Describe an algorithm to accept the values of A and B and swap them.
3. Describe an algorithm to accept the highest number from the user.
4. List all the steps used to search for 9 in the sequence $1, 3, 4, 5, 6, 8, 9, 11$ using a) a linear search. b) a binary search.
5. Describe an algorithm that inserts an integer x in the appropriate position into the list $a1, a2, \cdots, an$ of integers that are in increasing order.
6. Describe an algorithm based on the Selection sort for sorting the list $3, 2, 5, 4, 1, 9, 6, 8, 7, 2$.
7. Describe an algorithm that prints first n terms of the Fibonacci sequence.
8. Describe an algorithm that finds factorial of a non-negative integer.
9. Describe Euclidean algorithm to find GCD of given two integers.
10. Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$.
11. Show that n^2 is not $O(n)$.
12. Show that $x2 + 4x + 17$ is $O(x^3)$ but that x^3 is not $O(x^2 + 4x + 17)$.
13. Let k be a positive integer. Show that $1^k + 2^k + \cdots + n^k$ is $O(n^{k+1})$.
14. Towers of Hanoi for recursion.
15. What are the worst-case, average-case, and best-case time complexities, in terms of comparisons, of the algorithm that finds the smallest integer in a list of n integers by comparing each of the integers with the smallest integer found so far?
16. Drawing a graph, counting the degree of vertices and number of edges.
17. Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.
18. Determining whether the given graph is connected or not. Finding connected components of a graph. Finding strongly connected components of a graph. Finding cut vertices.
19. To determine whether the given graph is a tree. Construction of Binary Search Tree and applications to sorting and searching.

20. Spanning Trees. Finding Spanning Tree using Breadth First Search and/or Depth First Search.

21. Convert messages in to binary sequence using Hoffman's Algorithm.

SEMESTER IV

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

USMT401/UAMT401 CALCULUS IV

Unit I: Nested Interval theorem & Applications (15 Lectures)
Nested Interval theorem in $\mathbb{R}$. Applications of Nested Interval Theorem:

1. The set of real numbers is uncountable.
2. Decimal representation of a real number.
3. Bolzano Weierstrass Theorem: Every bounded sequence of real numbers has a convergent subsequence.
4. Intermediate Value theorem: Let $f : [a, b] \longrightarrow \mathbb{R}$ be a function continuous with $f(a)f(b) < 0$. Then $\exists\, c \in (a, b)$ such that $f(c) = 0$.
5. Heine-Borel theorem: Let $[a, b]$ be a closed and bounded interval and let $\{J_\alpha : \alpha \in \Lambda\}$ be a family of open intervals such that $[a, b] \subset \cup_{\alpha\in\Lambda} J\alpha$. Then there exists a finite subset such that $F \subseteq \Lambda$ such that $[a, b] \subset \cup_{\alpha\in F} J\alpha$.

Recommended Text Book for Unit I:
R.G. Bartle - D.R. Sherbet, *Introduction to Real analysis*, John Wiley & Sons.

Unit II: Riemann Integration (15 Lectures)
Definition of uniform continuity of a real valued function on a subset of $\mathbb{R}$. A continuous function on a closed and bounded interval is uniformly continuous (only statement).

Approximation of area; Upper/Lower Riemann sums and properties; Upper/Lower Riemann integrals; Definition of Riemann integral on a closed and bounded interval; Riemann's Criterion for Riemann integrability.

For $a < c < b$ $f \in \mathscr{R}[a, b]$ if and only if $f|_{[a,c]} \in \mathscr{R}[a, c]$ & $f|_{[c,b]} \in \mathscr{R}[c, b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

Properties of Riemann integrals:
i) $\lambda \in \mathbb{R}$ & $f, g \in \mathscr{R}[a, b] \Rightarrow f + g, \lambda f \in \mathscr{R}[a, b]$ & $\int_a^b (f + g) = \int_a^b f + \int_a^b g$, $\int_a^b \lambda f = \lambda \int_a^b f$

ii) $f \in \mathscr{R}[a,b] \Rightarrow |f| \in \mathscr{R}[a,b]$ and $| \int_a^b f | \leq \int_a^b |f|$
iii) $f \geq 0 \Rightarrow \int_a^b f \geq 0$.

$f \in \mathscr{C}[a,b] \Rightarrow f \in \mathscr{R}[a,b]$; every bounded function with finitely many discontinuities is Riemann integrable; monotone functions are Riemann integrable.

Recommended Text Books for Unit II:

1. R.G. Bartle -D.R. Sherbet, *Real analysis*, John Wiley & Sons.
2. T. Apostol, *Calculus Vol.2,* John Wiley.

Unit III: Indefinite and improper Riemann integrals, double integrals (15 Lectures)

1. Continuity of $F(x) = \int_a^x f(t)dt$ where $f \in \mathscr{R}[a,b]$. First and second Fundamental theorem of Calculus.

2. Mean value theorem for integrals. Integration by parts, Change of variable formula (statement only).

3. Improper integrals- type 1 and type 2; Absolute convergence of improper integrals; Comparison tests; Abels and Dirichlets tests (without proof).

4. Γ functions and their properties; β function $\beta(x,y)$, and relationship between β and Γ functions.

5. Double integrals: Definition of double integrals over rectangles, properties, double integrals over a bounded region.

Fubini theorem (without proof) - iterated integrals, double integrals as volume.

Application of double integrals: average value, area, moment, center of mass.

Double integral in polar form.
Reference for para 4 of unit III: W. Rudin, *Principles of Mathematical Analysis*, McGraw Hill.

Recommended Text Books for Unit III:

1. R.G. Bartle - D.R. Sherbet, *Introduction to Real analysis*, John Wiley & Sons.
2. J. E. Marsden- A. J. Tromba- A. Weinstein, *Basic multi-variable calculus*, Springer.

Additional reference books:

1. R. R. Goldberg, *Methods of Real Analysis*, Oxford and IBH, 1964.
2. Ajit Kumar & S. Kumaresan, *A Basic Course in Real Analysis,* CRC Press, 2014.
3. J. Stewart, *Calculus,* Brooke/Cole Publishing Co, 1994.
4. W. Rudin, *Principles of Mathematical Analysis*, McGraw Hill.

USMT402/UAMT402 Ordinary Differential Equations

Unit I: First order first degree differential equations (15 Lectures)

Definitions of: Differential Equation, Order and Degree of a Differential Equation, Ordinary Differential Equation (ODE), Partial Differential Equation, Linear ODE, non-linear ODE.

Definition of Lipschitz function, examples. Existence and Uniqueness Theorem for the differential equation $y' = f(x, y), y(x_0) = y_0$ where $f(x, y)$ is a continuous function satisfying Lipschitz condition $|f(x, y_1) - f(x, y_2)| \leq K|y_1 - y_2|$ on the strip $a \leq x \leq b \,\&\, y \in \mathbb{R}$ (statement only). Solve examples verifying the conditions of existence and uniqueness theorem.

Existence and Uniqueness Theorem for the solutions of a second order linear ODE:

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

with initial conditions $y(x_0) = y_0 \,\&\, y'(x_0) = y_1$ where $P(x), Q(x), R(x)$ are continuous functions on $[a, b]$ (statement only). Solve examples verifying the conditions of existence and uniqueness theorem.

Review of solution of homogeneous and non-homogeneous differential equations of first order and first degree.

Exact Equations: General Solution of Exact equations of first order and first degree, Necessary and sufficient condition for $M\,dx + N\,dy = 0$ to be exact. Non-exact equations. Rules for finding integrating factors (without proof) for non exact equations such as:

i) $\dfrac{1}{Mx + Ny}$ is an integrating factor if $Mx + Ny \neq 0$ & $Mdx + Ndy$ is homogeneous

ii) $\dfrac{1}{Mx - Ny}$ is an integrating factor if $Mx - Ny \neq 0$ & $Mdx - Ndy$ is of the form $f_1(x, y)y\,dx + f_2(x, y)x\,dy$

iii) a) $e^{\int f(x)dx}$ is an integrating factor if $N \neq 0$ & $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial N}\right)$ is a function of x alone, say $f(x)$

b) $e^{\int g(y)dy}$ is an integrating factor if $M \neq 0$ & $\dfrac{1}{M}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)$ is a function of y alone, say $g(y)$.

Linear and reducible to linear equations, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Linear Differential equations (15 Lectures) Existence and uniqueness theorems to be stated clearly when needed in the sequel.

Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equation. The use of known solutions to find the general solution of homogeneous equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.

The homogeneous equation which constant coefficient, auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.

Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Linear system of ODEs (15 Lectures)
Existence and uniqueness theorems to be stated clearly when needed in the sequel.

Study of homogeneous linear system of ODEs in two variables: Let $a_1(t)$, $a_2(t)$, $b_1(t)$, $b_2(t)$ be continuous real valued functions defined on $[a, b]$. Fix $t_0 \in [a, b]$. Then there exists a unique solution $x = x(t), y = y(t)$ valid throughout $[a, b]$ of the following system

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y,$$
$$\frac{dy}{dt} = a_2(t)x + b_2(t)y.$$

satisfying the initial conditions $x(t_0) = x_0$ & $y(t_0) = y_0$.

The Wronskian $W(t)$ of two solutions of a homogeneous linear system of ODEs in two variables, result: $W(t)$ is identically zero or nowhere zero on $[a, b]$. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables.

Explicit solutions of Homogeneous linear systems with constant coefficients in two variables, examples.

Recommended Text Books for Unit I and II:

1. G. F. Simmons, *Differential equations with applications and historical notes*, McGraw Hill.
2. E. A. Coddington, *An introduction to ordinary differential equations*, Dover Books.

Recommended Text Book for Unit III:
G. F. Simmons, *Differential equations with applications and historical notes*, McGraw Hill.

USMT403 INTRODUCTION TO COMPUTING AND PROBLEM SOLVING-II

Aim of this course:

to introduce Programming as a vehicle to test Algorithms *and enable the students to write their own Programs.*

Unit-I Problem solving strategies (15 lectures)

A. Problem Solving strategies

Problem analysis, formal definition of problem, Solution, top-down design, breaking a problem into sub problems, overview of the solution to the sub problems by writing step by step procedure (algorithm), flowcharts, pseudocodes

B. **Python programming language**:

1. Variables, expressions and statements Values and types:int, float and str
   Variables: assignment statements, printing variable values, types of variables.
2. Operators, operands and precedence:+, -, /, *, **, %
   PEMDAS(Rules of precedence)
3. String operations: + : Concatenation, * : Repetition
4. Boolean, Comparison and Logical operators:Boolean operator: ==
   Comparison operators: ==, !=, >, <, >=, <=
   Logical operators: and, or, not
   Mathematical functions: sin, cos, tan, log, sqrt etc.
   Keyboard input: raw_input statement

Unit-II: Iterations and Conditional statements (15 lectures)

A. Conditional and alternative statements, Chained and Nested Conditionals:
if, if-else, if-elif-else, nested if, nested if-else
looping statements such as while, for etc
Tables using while.

B. Functions:
Calling functions: type, id
Type conversion:int, float, str
Type coercion
Composition of functions
User defined functions, Parameters and arguments

Unit-III Strings (15 lectures)

A. Elementary Python Graphics

B. Strings and Lists in Python
Strings: Compound data types, Length(len function)
String traversal: Using while statement, Using for statement
Comparison operators(>, <. ==)
Lists and List operations
Use of range function Accessing list elements
List membership and for loop
List operations
List updation: addition, removal or updation of elements of a list

Recommended Text Books

1. Downey, A. et al., *How to think like a Computer Scientist: Learning with Python*, John Wiley, 2015.
2. Goel, A., *Computer Fundamentals*, Pearson Education.
3. Lambert K. A., *Fundamentals of Python - First Programs*, Cengage Learning India, 2015.
4. Rajaraman, V., *Computer Basics and C Programming*, Prentice-Hall India.

Additional References Books

1. Barry, P., *Head First Python,* O Reilly Publishers.
2. Dromy, R. G., *How to solve it by Computer*, Pearson India.
3. Guzdial, M. J., *Introduction to Computing and Programming in Python*, Pearson India.
4. Perkovic, L., *Introduction to Computing Using Python*, 2/e, John Wiley, 2015.
5. Sprankle, M., *Problem Solving & Programming Concepts*, Pearson India.
6. Venit, S. and Drake, E., *Prelude to Programming: Concepts & Design*, Pearson India.
7. Zelle, J., *Python Programming: An Introduction to Computer Science*, Franklin, Beedle & Associates Inc.

USMTP04/UAMTP04 Practicals

A. Practical for USMT401/UAMT401:

(1) Calculation of upper sum, lower sum and Riemann integral.

(2) Problems on properties of Riemann integral.

(3) Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.

(4) Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.

(5) Sketching of regions in $\mathbb{R}^2$ and $\mathbb{R}^3$, graph of a function, level sets, conversions from one coordinate system to another.

(6) Double integrals, iterated integrals, applications to compute average value, area, moment, center of mass and evaluating double integrals $\int\int f(x,y)\,dxdy$ on a rectangle Q similar to following :

(a) $Q = [0,1] \times [0,1]$ and $f(x,y) = \begin{cases} 1-x-y & \text{if } x+y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$

(b) $Q = [0,1] \times [0,1]$ and $f(x,y) = \begin{cases} x^2+y^2 & \text{if } x^2+y^2 \leq 1, \\ 0 & \text{otherwise.} \end{cases}$

(c) $Q = [1,2] \times [1,4]$ and $f(x,y) = \begin{cases} (x^2+y^2)^{-1} & \text{if } x \leq y \leq 2x, \\ 0 & \text{otherwise.} \end{cases}$

(d) $Q = [0,1] \times [0,1]$ and $f(x,y) = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{otherwise.} \end{cases}$

(e) $Q = [0,\pi] \times [0,\pi]$ and $f(x,y) = |\cos(x+y)|$.

(7) Applications of Nested interval theorem.

B. Practical for USMT402/UAMT402:

1) Solving exact and non exact equations.
2) Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3) Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4) Solving equations using method of undetermined coefficients and method of variation of parameters.
5) Solving second order linear ODEs.
6) Solving a system of first order linear ODEs.

C. Practical for USMT403:

For the Practicals of USMT403, Python version 2.7.9 shall be used by all colleges.

Suggested Practicals for Unit I:
Using Algorithm and flowcharts:

a) find factorial of non negative number n
b) find largest among a given set of numbers
c) generating Fibonacci sequences.
d) arrange n numbers in increasing order.

Installing and setting up of Python IDLE interpreter.

Suggested Practical for Unit II:
Suggested simple exercises:

1. Type Python statements to check the data types of the following:
   a) "Hello World!"
   b) 1532
   c) -265.3665
2. Type Python statements to assign similar to the following values to the variables, check variable data types and print the variable values:
   a) "Hello World!"
   b) 1532
   c) -265.3665
3. Using Python check output of the statements similar to following :
   a) >>> 1+1
   b) >>> 17
   c) >>> message="Python"
      >>> message
   d) >>> x=55
      >>> x/60
   e) >>> 12**2/(5-1)
   f) >>> "Hello"*4
   g) >>> "Hello "+"World!"
   h) >>> id(5)
   i) >>> id("Hello")
   j) >>> int(-17.3256)
   k) >>> str(-17.3256)
   l) >>> float(55/60)
4. Type a Python code to accept an integer and check whether it's even , odd or prime.
5. Using Python evaluate Mathematical expressions similar to the following:
   a) $\sqrt{2}$
   b) $\sin(\pi/2)$
   c) $\cos(\theta + \pi/2)$ where θ is entered by the user.
   d) $e^{(\log_{10}(x))}$ where x is entered by the user.
6. Circles, lines drawing.
7. Roots of quadratic equations.

Suggested Practical for Unit III:

1. Type a python code to display integers ranging from 1 to n.(value of n is entered by the user)

2. Type a Python code that displays integers 1 to 10 and their squares in the table format using while statement.

3. Type Python statements to assign the value"Hello World!" to the variable message and print the following:

   a) first letter of the variable message.

   b) length of the variable message.

   c) last letter of the variable message.

   d) each letter from the string using while statement.

   e) each letter from the string using for statement.

4. Check output of the following statements with respect to Python:

   a) >>> range(1:5)

   b) >>> range(10)

   c) >>> range(1, 10, 2)

   d) horsemen = ["war", "famine", "pestilence", "death"]
      $i = 0$
      while $i < 4$:
      print horsemen$[i]$
      $i = i + 1$

   e) >>> $a = [1, 2, 3]$
      >>> $b = [4, 5, 6]$
      >>> $c = a + b$
      >>> print c

5. Type a Python code to display all odd numbers from 1 to 20 using lists and for statement.

6. Define a 3×3 matrix using lists in Python. Type a code to display each row of the matrix and each element of the matrix.

7. Type a Python code to define a function for swapping the two variable values using tuples.

8. Check output of the following with respect to python:

   a)
      >>> eng2sp = {}
      >>> eng2sp['one'] = 'uno'
      >>> eng2sp['two'] = 'dos'
      >>> print eng2sp

   b)
      >>> eng2sp = {'one': 'uno', 'two': 'dos', 'three': 'tres'}
      >>> print eng2sp

   c)
      >>> inventory={'apples':430,'bananas':312,'oranges':525,'pears':217}

```
>>> del inventory['pears']
>>> print inventory
```

d)

```
>>> inventory={'apples':430,'bananas':312,'oranges':525,'pears':217}
>>> inventory['pears'] = 0
>>> print inventory
```

e)

```
>>> inventory={'apples':430,'bananas':312,'oranges':525,'pears':217}
>>> len(inventory)
```

9. Type a Python statement to create a file object test.dat.
10. Type a Python statement to put data "Hello World!" in the file test.dat.
11. Type a Python statement to close the file test.dat.
12. Type a Python statement to read the file test.dat.
13. Type a Python statement to read the file test.dat in a directory named test, which resides in share, which resides in user, which resides in the top-level directory of the system, called /.
14. Type a Python code to prompt the user for the name of a file and then try to open it. If the file doesn't exist, we don't want the program to crash.(Use try and except statements.)
15. Write Python programs for the following:
    (a) To solve quadratic equation.
    (b) To find factorial of non-negative number n.
    (c) to find largest among a given set of n numbers $(n \geq 3)$.
    (d) To find nth term of Fibonacci sequence.
    (e) To find area of circle.
    (f) To find sum, maximum and minimum of n numbers.
    (g) To arrange n numbers in increasing order.
    (h) To arrange n names in alphabetic order.
    (i) to find g.c.d. of two integers
    (j) recursion for Tower of Hanoi
    (k) Selection sort, merge sort.

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Scheme of Examination

I. Semester End Theory Examinations:
There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT301/UAMT301, USMT302/UAMT302, USMT303 of Semester III and USMT401/UAMT401, USMT402/UAMT402, USMT403 of semester IV to be conducted by the University.

1. Duration: The examinations shall be of 3 Hours duration.

2. Theory Question Paper Pattern:
   a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q3, Q4 shall be of 20 marks, each based on the units I, II , III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
   b) All the questions shall be compulsory. The questions Q2,Q3,Q4,Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
   c) The questions Q2,Q3,Q4,Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
   (a) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. Semester End Examinations Practicals:
At the end of the Semesters III & IV, Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III & IV, Practical examinations of two hours duration and 100 marks shall be conducted for the courses UAMTP03, UAMTP04.

In semester III, the Practical examinations for USMT301/UAMT301 and USMT302/UAMT302 are held together by the college. The Practical examination for USMT303 is held separately by the college.

In semester IV, the Practical examinations for USMT401/UAMT401 and USMT402/UAMT402 are held together by the college. The Practical examination for USMT403 is held separately by the college.

For the Practicals of USMT403, Python version 2.7.9 shall be used by all colleges.

Paper pattern: The question paper shall have three parts A,B, C. Every part shall have three questions of 20 marks each. Students to attempt any two question from each part.

Practical Course	Part A	Part B	Part C	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT303	120	3 hours
UAMTP03	Questions from UAMT301	Questions UAMT302	-	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP04	Questions from UAMT401	Questions from UAMT402	-	80	2 hours

Marks for Journals and Viva: For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402 and USMT403

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester III & IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal.

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